The Motion of a Thermally Conducting Sphere in a Rarefied Gas, I. Low Speed Case

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An investigation has been made of the validity of the perfectly conducting sphere model used in the calculation of drag forces acting on spheres moving in rarefied gases. This assumption requires that the frictional heat generated by the motion of the sphere is conducted so rapidly through the material that the surface temperature is everywhere constant. In turn this affects the energy of the atoms reflected from the surface of the body and hence the drag experienced by it. Instead, therefore, of making this *a priori* assumption, we allow the sphere to have an arbitrary thermal conductivity. We then solve the heat conduction equation in the sphere and relate it to the external gas conditions by computing the heat transfer rate caused by gas atom collisions. The theory so developed is applicable for arbitrary speed but, for simplecity in this introductory paper, we obtain some analytical results for speeds very much less than Mach one.

Our conclusions indicate that the effects of finite conduction on the drag forces are generally small, even when the sphere is a thin shell with a non-conducting interior. Indeed, it is not difficult to show that in going from a perfect thermal conductor to a perfect thermal insulator the drag force only increases by about 3%; nevertheless, in some situations this may well be important and intermediate cases will have to include the correction term. More significantly, however, the surface temperature on the sphere is shown to depend on the conductivity to a much greater degree, with the leading face being appreciably hotter than the trailing one.

The general conclusion is that for most practical problems involving small particles in the Knudsen regime, moving at appreciably sub-sonic speeds, the assumption of the perfect thermal conductor is a good one.

1. Introduction

A basic assumption employed in the calculation of drag forces on bodies moving through rarefied gases is that of a uniform surface temperature. Indeed, until recently [1] even the variation of this surface temperature with speed had been neglected. Investigation of the limitations of this latter effect indicated that significant corrections to the drag coefficient above Mach one were required. Similarly it may be asked why the variation of the surface temperature with position has not been taken into account. On physical grounds it certainly appears that, as the velocity of the body increases, the leading face will experience a greater momentum exchange from the collision rate than the trailing one and hence become hotter. In addition to the variation of the surface temperature, we can also expect a variation of the force over the body; thus whilst the drag coefficient is a measure of the average value of this force it does not enable local stresses to be evaluated which are certain to be important at high speeds and possibly significant at sub-sonic speeds.

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The basic assumption of a constant surface temperature appears to have originated from the work of Epstein [2] who by not unreasonable, but certainly qualitative, arguments showed that the body (in his case a sphere) could be considered as a perfect thermal conductor provided the sphere radius was very much less than 35 mean free paths of atoms in the surrounding gas. Since the free molecule theory on which rarefied flow characteristics are based is only valid for sphere radii less than about two or three mean free paths Epstein's result indicates that the perfect thermal conduction theory is valid. However, no rigourous proof has ever been provided and moreover Epstein's results were obtained only for low speed problems, i.e. Mach numbers very much less than unity. Thus there are two questions left unanswered: (1) how can Epstein's assumption be proved rigorously and (2) does the assumption hold for high Mach numbers?

It is the purpose of the present paper to investigate the first of these assumptions with a companion paper reporting on the second aspect. This subdivision is convenient since the low speed case is amenable to a relatively simple analytical treatment whereas the high Mach number case requires some extensive computation.

The form of the paper, therefore, will consist of deducing the differential force acting on the sphere and also the differential heat transfer rate. These basic quantities are then used as boundary conditions to solve the thermal conduction equation in the sphere and hence the temperature profile in the sphere is obtained. The treatment will be for arbitrary speed but results for low speeds only will be given, the general case being dealt with in Part II. It is assumed that a simple diffuse, perfectly accommodating surface scattering law describes the gas-surface interaction, although this restriction can be removed at the expense of additional algebra.

2. Basic Theory

In all that follows we shall assume a spherical body although generalisation to other shapes is straightforward if sometimes tedious. Consider therefore Fig. 1 which represents a sphere of radius 'a', in the free molecule regime, with an incident gas distribution $f_0(\boldsymbol{v})$ given by

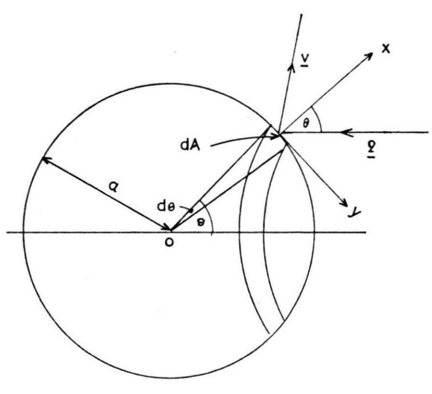

Fig. 1. q denotes the macroscopic velocity of the gas stream incident on the sphere. dA is the annulus on which momentum and energy balances are made.

$$f_0(\boldsymbol{v}) = n_0 (\beta_0/\pi)^{3/2} \exp\{-\beta_0 (\boldsymbol{v}-\boldsymbol{q})^2\} \quad (1)$$

where n_0 is the number density, $\boldsymbol{q}$ the macroscopic flow velocity and $\beta_0 = m/2kT_0$, T_0 being the gas temperature.

We consider a surface element $\mathrm{d}A$ on the sphere and assume that complete accommodation occurs such that incident particles are re-emitted isotropically in a Maxwellian distribution with a temperature equal to the surface temperature T_w. Thus the reflected molecules have the following velocity distribution:

$$f_\mathrm{w}(\boldsymbol{v}) = n_\mathrm{w} (\beta_\mathrm{w}/\pi)^{3/2} \exp\{-\beta_\mathrm{w} v^2\} \quad (2)$$

where $\beta_\mathrm{w} = m/2kT_\mathrm{w}$.

The number density n_w is obtained by imposing the condition of particle conservation at the surface, i.e.

$$\int_{-\infty}^{\infty} \mathrm{d}v_y \int_{-\infty}^{\infty} \mathrm{d}v_z \int_{0}^{\infty} \mathrm{d}v_x\, \boldsymbol{n}\cdot\boldsymbol{v}\, f_\mathrm{w}(\boldsymbol{v}) = \int_{-\infty}^{\infty} \mathrm{d}v_y \int_{-\infty}^{\infty} \mathrm{d}v_z \int_{0}^{-\infty} \mathrm{d}v_x\, \boldsymbol{n}\cdot\boldsymbol{v}\, f_0(\boldsymbol{v}) \quad (3)$$

where $\boldsymbol{n}$ is the unit normal at $\mathrm{d}A$ pointing out of the body and we have defined the velocity co-ordinate system such that $\boldsymbol{n}\cdot\boldsymbol{v} = v_x$. Thus v_y is in the direction y as indicated and v_z is directed out of the plane of the paper.

Evaluating the integrals, we obtain

$$n_\mathrm{w} = n_0 \left(\frac{\beta_\mathrm{w}}{\beta_0}\right)^{1/2} \{\exp\{-\beta_0 q_x^2\} - \sqrt{\pi}\, \beta_0^{1/2} q_x [1 - \mathrm{erf}\,(\beta_0^{1/2} q_x)]\} \quad (4)$$

where it should be noted that in our co-ordinate system $q_x = -q\cos\theta$, $q_y = -q\sin\theta$, $q_z = 0$.

2.1 The Force on the surface element

The net force $\mathrm{d}\boldsymbol{F}$ on the element $\mathrm{d}A$ is given by

$$\frac{1}{m}\frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}A} = \int_{-\infty}^{\infty} \mathrm{d}v_z \int_{-\infty}^{\infty} \mathrm{d}v_y \int_{0}^{\infty} \mathrm{d}v_x\, v_x \boldsymbol{v}\, f_\mathrm{w}(\boldsymbol{v}) - \int_{-\infty}^{\infty} \mathrm{d}v_z \int_{-\infty}^{\infty} \mathrm{d}v_y \int_{0}^{-\infty} \mathrm{d}v_x\, v_x \boldsymbol{v}\, f_0(\boldsymbol{v}) \quad (5)$$

which if we write $\boldsymbol{v} = \boldsymbol{i}\, v_x + \boldsymbol{j}\, v_y + \boldsymbol{k}\, v_z$ leads after integration to

$$\frac{1}{m}\frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}A} = \frac{1}{4}\boldsymbol{i}\left\{\frac{n_\mathrm{w}}{\beta_\mathrm{w}} - \frac{2}{\sqrt{\pi}}\frac{n_0 q_x}{\beta_0^{1/2}} \exp\{-\beta_0 q_x^2\} + \frac{n_0}{\beta_0}[1 + 2\beta_0 q_x^2]\,[1 - \mathrm{erf}(\beta_0^{1/2} q_x)]\right\} - \boldsymbol{j}\,\frac{q_y n_0}{2\beta_0^{1/2}}\left\{\frac{1}{\sqrt{\pi}} \exp\{-\beta_0 q_x^2\} - \beta_0^{1/2} q_x [1 - \mathrm{erf}(\beta_0^{1/2} q_x)]\right\}. \quad (6)$$

The component of the force in the direction of motion is given by $\hat{\boldsymbol{q}}\cdot\mathrm{d}\boldsymbol{F} \equiv \mathrm{d}F_q$. Thus with $\hat{\boldsymbol{q}}\cdot\boldsymbol{i} = -\cos\theta$ and $\hat{\boldsymbol{q}}\cdot\boldsymbol{j} = -\sin\theta$, we have after using Eq. (4)

$$\frac{1}{m}\frac{\mathrm{d}F_q}{\mathrm{d}A} = -\frac{1}{4}\cos\theta\left[\frac{n_0}{(\beta_0\beta_\mathrm{w})^{1/2}}\{\exp\{-\beta_0 q^2\cos^2\theta\} + \sqrt{\pi}\,\beta_0^{1/2} q\cos\theta\,[1+\mathrm{erf}(\beta_0^{1/2} q\cos\theta)]\}\right.$$
$$+\frac{n_0}{\beta_0}\left\{\frac{2}{\sqrt{\pi}}\beta_0^{1/2} q\cos\theta\exp\{-\beta_0 q^2\cos^2\theta\} + [1+2\beta_0 q^2\cos^2\theta]\,[1+\mathrm{erf}(\beta_0^{1/2} q\cos\theta)]\right\}\Bigg]$$
$$-\frac{n_0}{2\beta_0}\beta_0^{1/2} q\sin^2\theta\left[\frac{1}{\sqrt{\pi}}\exp\{-\beta_0 q^2\cos^2\theta\} + \beta_0^{1/2} q\cos\theta\,[1+\mathrm{erf}(\beta_0^{1/2} q\cos\theta)]\right]. \tag{7}$$

We now introduce the symbols $s = \beta_0^{1/2} q$ and $\mu = \cos\theta$, whence Eq. (7) becomes

$$-\frac{4}{m}\frac{\beta_0}{n_0}\frac{\mathrm{d}F_q}{\mathrm{d}A} = \mu\left(\frac{T_\mathrm{w}}{T_0}\right)^{1/2}\{e^{-s^2\mu^2} + \sqrt{\pi}\,s\,\mu\,[1+\mathrm{erf}(s\,\mu)]\}$$
$$+\frac{2}{\sqrt{\pi}}\,s\,e^{-s^2\mu^2} + (1+2s^2)\,\mu[1+\mathrm{erf}(s\,\mu)]. \tag{8}$$

Integration of this expression over the surface of the sphere with $\mathrm{d}A = 2\pi a^2\sin\theta\,\mathrm{d}\theta$ leads, for the case of T_w independent of μ to the value of the net force on the sphere given by previous workers in the field[3, 4]. Moreover, given T_w, Eq. (8) enables the variation of the force over the surface of the body to be studied and hence the magnitude of any stresses arising from gradients in the force.

The basic unknown therefore is the surface temperature T_w. To obtain this quantity it is necessary to compute the net heat transfer due to collisions at the surface element $\mathrm{d}A$.

2.2 Heat Transfer to a surface element

If we denote by $\mathrm{d}Q$ the net amount of heat transferred to $\mathrm{d}A$ per unit time by collisions with the surface, then by the usual reasoning, it may be written

$$\frac{\mathrm{d}Q}{\mathrm{d}A} = \int_{-\infty}^{\infty}\mathrm{d}v_y\int_{-\infty}^{\infty}\mathrm{d}v_z\int_{0}^{\infty}\mathrm{d}v_x\,\boldsymbol{n}\cdot\boldsymbol{v}\,(\tfrac{1}{2}m v^2)\,f_\mathrm{w}(\boldsymbol{v}) - \int_{-\infty}^{\infty}\mathrm{d}v_y\int_{-\infty}^{\infty}\mathrm{d}v_z\int_{0}^{-\infty}\mathrm{d}v_x\,\boldsymbol{n}\cdot\boldsymbol{v}\,(\tfrac{1}{2}m v^2)\,f_0(\boldsymbol{v}). \tag{9}$$

Inserting the expressions for f_w and f_0 and carrying out the integrations, we find that

$$\frac{2}{m}\frac{\mathrm{d}Q}{\mathrm{d}A} = \frac{n_\mathrm{w}}{\sqrt{\pi}\,\beta_\mathrm{w}^{3/2}} - \frac{n_0}{\sqrt{\pi}\,\beta_0^{3/2}}\left\{\tfrac{1}{2}(1+\beta_0 q_x^2)\exp\{-\beta_0 q_x^2\} - \frac{\sqrt{\pi}}{4}[1-\mathrm{erf}(\beta_0^{1/2} q_x)]\,q_x^3)\,[1-\mathrm{erf}(\beta_0^{1/2} q_x)]\right.$$
$$\left.+\tfrac{1}{2}(1+\beta_0 q_y^2)\left(\exp\{-\beta_0 q_x^2\} - \frac{\sqrt{\pi}}{2}\beta_0^{1/2} q_x[1-\mathrm{erf}(\beta_0^{1/2} q_x)]\right)\right\}. \tag{10}$$

Using the values for q_x and q_y and changing to the μ and s variables, we obtain

$$\frac{2\beta_0^{3/2}}{m\,n_0}\frac{\mathrm{d}Q}{\mathrm{d}A} = \frac{T_\mathrm{w}}{T_0}\left\{\frac{e^{-s^2\mu^2}}{\sqrt{\pi}} + s\,\mu\,[1+\mathrm{erf}(s\,\mu)]\right\} - \left\{\frac{e^{-s^2\mu^2}}{\sqrt{\pi}}(1+\tfrac{1}{2}s^2) + \tfrac{1}{2}s\,\mu\,(s^2+\tfrac{5}{2})\,[1+\mathrm{erf}(s\,\mu)]\right\} \tag{11}$$
$$\equiv (T_\mathrm{w}/T_0)\,f(s,\mu) - g(s,\mu). \tag{12}$$

Given T_w we are now able to calculate the variation of heat transfer to the surface of the body at any speed. It should be noted that in this derivation, and throughout, we have neglected losses due to radiation.

3. The Surface Temperature

At equilibrium, the net heat transfer to the sphere will be zero. Thus we can write with complete certainty that

$$\int_A \mathrm{d}Q = 0. \tag{13}$$

In the past, two limiting cases have been considered: (1) the perfect thermal conductor, which implies that the surface temperature T_w is everywhere equal and (2), the perfect thermal nonconductor, which maintains that any heat transferred to the element $\mathrm{d}A$ is not transported away and consequently $\mathrm{d}Q = 0$ at each point. To put it otherwise, the local temperature will increase or decrease until the energy carried off by reflected atoms is just balanced by that brought by incident ones. In the general case this is ensured in a global sense by the condition (13).

For the perfect conductor, with T_w independent of μ, condition (13) shows by the use of Eq. (12) that the surface temperature T_w is given by

$$T_w(s)/T_0 = \int_{-1}^{1} d\mu\, g(s,\mu) \Big/ \int_{-1}^{1} d\mu\, f(s,\mu)\,. \quad (14)$$

This integral has been evaluated in Ref. 1 and shows that a considerable increase of the surface temperature can be expected as the speed increases. In addition, the effect on the drag, via Eq. (8) is also substantial.

In the case of the perfect thermal non-conductor, the condition $dQ = 0$ locally, leads to

$$T_w(s,\mu)/T_0 = g(s,\mu)/f(s,\mu) \quad (15)$$

which indicates a variation of surface temperature not only with speed but also as a function of position on the surface of the sphere. In many ways this result is more realistic physically since it is to be expected that the leading face will become hotter than the trailing one. Naturally, however, the magnitude of the temperature difference will depend upon the thermal conductivity of the material of the sphere. The drag also will be affected by the dependence of T_w on μ since this factor enters Equation (8). Unfortunately, when (15) is used in (8) the integrations can no longer be carried out analytically. However, preliminary numerical integrations indicate that whilst T_w varies considerably with position for speeds above $s=1$, the effect on the drag coefficient at $s=1$, is only 1.35% and becomes smaller as s increases. Thus, as far as drag is concerned, the major effect of the conductivity of the sphere occurs at low speeds. On the other hand, the variation of the drag over the surface of the sphere is very considerable, irrespective of the surface temperature. However, as stated in the Introduction, we shall examine high Mach number effects in a companion paper.

Between the two limits discussed above lies the true situation. Thus it will be necessary to solve the thermal conduction equation in the sphere and to use Eq. (12) as a boundary condition.

3.1 The general case

For the sake of generality we shall consider a hollow sphere the interior of which is a perfect thermal non-conductor and the shell of which has an arbitrary thermal conductivity. The inner radius is 'b' and the outer one is 'a'.

In the shell the temperature $T(r,\mu)$ will satisfy Laplace's equation which in the appropriate coordinates of Fig. 1 we may write as

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial T}{\partial \mu}\right) = 0\,. \quad (16)$$

The boundary condition at $r=b$ is

$$\left.\frac{\partial T(r,\mu)}{\partial r}\right|_{r=b} = 0 \quad (17)$$

whilst at $r=a$ we have

$$\frac{dQ}{dA} = -K \left.\frac{\partial T(r,\mu)}{\partial r}\right|_{r=a}, \quad (18)$$

K being the thermal conductivity. Using Eqs. (11) and (12) we may rewrite (18) as follows:

$$T(a,\mu) f(s,\mu) - T_0\, g(s,\mu) = -\beta \left.\frac{\partial T(r,\mu)}{\partial r}\right|_{r=a}. \quad (19)$$

In constructing this boundary condition we have noted that $T_w \equiv T(a,\mu)$, and have introduced the parameter β such that

$$\beta = \frac{2 K T_0}{m\, n_0}\left(\frac{m}{2 k T_0}\right)^{3/2}. \quad (20)$$

We note that β has the dimensions of length.

The general solution of Eq. (16) can be written

$$T(r,\mu) = \sum_{l=0}^{\infty}\left\{A_l \left(\frac{r}{a}\right)^l + B_l\left(\frac{a}{r}\right)^{l+1}\right\} P_l(\mu) \quad (21)$$

where $P_l(\mu)$ are the Legendre polynomials.

Using boundary condition (17) enables us to relate B_l to A_l viz:

$$B_l = A_l \frac{l}{l+1}\left(\frac{b}{a}\right)^{2l+1}. \quad (22)$$

Thus $T(r,\mu)$ can now be written as

$$T(r,\mu) = \sum_{l=0}^{\infty} A_l \left\{\left(\frac{r}{a}\right)^l + \frac{l}{l+1}\left(\frac{b}{a}\right)^{2l+1}\left(\frac{a}{r}\right)^{l+1}\right\} P_l(\mu)\,. \quad (23)$$

Inserting (23) into the other boundary condition does not lead to an explicit solution for the A_l. However, if Eq. (19) is multiplied by $P_m(\mu)$ and integrated over $\mu(-1,1)$ we can arrive at the following set of algebraic equations for the coefficients

A_l, with $y = b/a$

$$\sum_{l=0}^{\infty} A_l \left[\left\{ 1 + \frac{l}{l+1} y^{2l+1} \right\} f_{ml} + \frac{\beta}{a} \frac{2m}{2m+1} \{1 - y^{2m+1}\} \delta_{ml} \right] = g_m T_0 \quad (24)$$

where $m = 0, 1, 2, \ldots$ and g_m and f_{ml} are defined by

$$g_m = \int_{-1}^{1} d\mu \, P_m(\mu) g(s, \mu) \quad (25)$$

$$f_{ml} = \int_{-1}^{1} d\mu \, P_m(\mu) P_l(\mu) f(s, \mu) \, . \quad (26)$$

By taking a sufficient number of terms in the sum we can evaluate the A_l to any desired accuracy. For large values of s which are associated with a rapid variation of $T(r, \mu)$ with μ it will be necessary to employ the services of a digital computer. On the other hand to examine the behaviour for $s \ll 1$ it is only necessary to take two terms in the expansion. When this is done, and terms of order s^2 discarded, it is readily found that $A_0 = T_0$ and

$$A_1/T_0 = (\sqrt{\pi}\, s/4) \Big/ \left(1 + \tfrac{1}{2} y^3 + \frac{\sqrt{\pi}\,\beta}{a} (1 - y^3) \right) . \quad (27)$$

The surface temperature of the sphere may therefore be written

$$\frac{T_w(\mu)}{T_0} \equiv \frac{T(a, \mu)}{T_0} = 1 + \frac{(\sqrt{\pi}\, s\, \mu/4)}{\left[1 + \dfrac{\sqrt{\pi}\,\beta}{a} \dfrac{(1 - y^3)}{(1 + \frac{1}{2} y^3)} \right]} . \quad (28)$$

It follows from this formula that for a sphere which is a perfect thermal conductor, i.e. $\beta = \infty$, the surface temperature is constant. For the perfect non-conductor or any intermediate state the surface temperature is a function of position.

Having obtained this first approximation to the surface temperature we may use it to find the value of dQ. Thus to the designated order of s, we can write

$$\frac{2 \beta_0^{3/2}}{m n_0} \frac{dQ}{dA} = - \frac{s \mu}{4} \frac{\left[\dfrac{\sqrt{\pi}\,\beta}{a} (1 - y^3) \right]}{\left[1 + \frac{1}{2} y^3 + \dfrac{\sqrt{\pi}\,\beta}{a} (1 - y^3) \right]} \quad (29)$$

which shows that $\int dQ = 0$ as required. In addition, however, we note that $dQ < 0$ for the leading face, $\mu > 0$, and $dQ > 0$ for the trailing face, $\mu < 0$. Thus heat is conducted through the shell of the hollow sphere to balance the net heat flow. A further point to note is that $y = 1$, i.e. $b = a$, is equivalent to $\beta = 0$ as we would expect in view of the nature of the central core.

3.2 The drag on the sphere

Expanding Eq. (8) to the same order as for the temperature, it is found that

$$- \frac{4 \beta_0}{m n_0} \frac{dF_q}{dA} = \frac{2s}{\sqrt{\pi}} + 2\mu + \frac{2s}{\sqrt{\pi}} \mu^2 + \sqrt{\pi}\, s\, \mu^2 \left(1 + (1/8) \Big/ \left[1 + \frac{\sqrt{\pi}\,\beta}{a} \frac{(1 - y^3)}{(1 + \frac{1}{2} y^3)} \right] \right)$$

which after integration over the surface and re-arrangement gives for the net force on the sphere the value

$$-F_q = \frac{4}{3} m n_0 A \left(\frac{8 k T_0}{\pi m} \right)^{1/2} q \left\{ 1 + \frac{\pi}{8} + \frac{\pi}{64} \frac{1}{\left[1 + \dfrac{\sqrt{\pi}\,\beta}{a} \dfrac{(1 - y^3)}{(1 + \frac{1}{2} y^3)} \right]} \right\} . \quad (30)$$

We note that for a perfect thermal conductor where $\beta = \infty$ the factor in curly brackets becomes $1 + \pi/8$ and hence it reduces to the accepted value for a solid sphere, conversely, for a perfect non-conductor, $\beta = 0$ and the curly bracket becomes $1 + 9\pi/64$ as predicted by Epstein [2] and using another method by Williams [1]. We also note, as expected, that when $b = a$, $y = 1$, that the perfect non-conducting limit is given.

Formulae (28) and (30) provide us with general expressions for calculating drag and temperature profile for the general case of a hollow sphere whose shell has an arbitrary thermal conductivity.

4. Discussion and Conclusions

The purpose of this investigation has been to assess the validity of the assumption by which we regard bodies moving in rarefied gases as having infinite thermal conductivity. In addition we have computed temperatures within the body. Our analysis indicates that correction terms are present and it therefore remains to assess the conditions under which these corrections are significant. All calculations have been carried out for velocities very much less than $(2\,k\,T/m)^{1/2}$, although the general formalism can be extended to arbitrary speeds.

Before inserting any numbers, it is important to note that the conductivity of the material and the shell thickness can have only a very small effect upon the drag force. Thus for $\beta = 0$ the quantity in curly brackets in Eq. (30) is 1.442 whilst for $\beta = \infty$ it is 1.393. The temperature, however, is rather more sensitive to the value of β and the shell thickness; even so at the low velocities considered here the temperature differential is unlikely to be large. Using the values given by Epstein, which correspond to homogeneous spheres of olive oil in air, we calculate that $\sqrt{\pi}\,\beta \cong 8\,l$, where l is the mean free path of gas atoms in the surrounding gas. For copper, $\sqrt{\pi}\,\beta$ would be about 2000 times larger. However, considering the case for which $a = 4\,l$, we obtain from Eq. (30) with $y = 0$, the value of 1.409 for the quantity in the curly brackets. On the other hand, the fractional temperature difference between leading and trailing faces is $\sqrt{\pi}\,s/6$. For $s = 0.1$ this amounts to around 3%.

In view of the above analysis it is clear that for most practical purposes the assumption introduced by Epstein of treating the body as perfectly conducting is a good one. It is unlikely to have any measurable effect on small particle behaviour such as aerosols, dusts or fogs under low velocity conditions. However, the presence of temperature differentials should be borne in mind, particularly for highly volatile materials.

The general theory described above is likely to be of some value for high speed bodies made from composite materials. In such cases the internal temperatures are important parameters of the motion and our general theory can easily be modified to account for internal structure. In particular, high altitude satellites with thin shells could be subjected to severe temperature gradients and stresses. We shall investigate this aspect of the problem in the second part of this paper and discuss its importance in the field of ablation and accretion.

Appendix

Extension to a two region sphere

Consider a sphere composed of two concentric regions in each of which the conductivity is finite. By solving Laplace's equation in the central region and using the boundary conditions

$$T_1(b) = T_2(b)$$

and

$$\beta_1 \frac{\partial T_1}{\partial r}\bigg|_b = \beta_2 \frac{\partial T_2}{\partial r}\bigg|_b$$

we may obtain the temperature at the surface of the sphere and hence obtain the force on it. Our calculation shows that the force is modified to the extent that the denominator in the third term of Eq. (30), in the curly brackets, is replaced by

$$1 + \frac{\sqrt{\pi}\,\beta_1}{a} \frac{\left(1 - 2\,y^3\,\dfrac{\beta_1 - \beta_2}{2\,\beta_1 + \beta_2}\right)}{\left(1 + y^3\,\dfrac{\beta_1 - \beta_2}{2\,\beta_1 + \beta_2}\right)}\,.$$

This reduces to the previous results when the limiting cases are considered.

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